5.3b Definite Integrals and Antiderivatives

Rules for Definite Integrals

Nov 13-4:58 PM

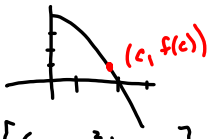
Average (Mean) Value

Nov 13-5:05 PM

Find the average value of f(x) = 4-x^2 on [0,3]. Does *f* actually take on this value at some point on the given interval? If so, where?

yes

$$\bar{y} = \frac{1}{3-0}\int_0^3 4-x^2\,dx$$



$$\bar{y} = \frac{1}{3}\left[4x - \frac{x^3}{3}\Big|_0^3\right] = \frac{1}{3}\left[\left(4\cdot 3 - \frac{3^3}{3}\right) - 0\right]$$

$f(c) = \bar{y} = 1$ {average value}

$1 = 4 - x^2$

$c = x = \sqrt{3}$ {the value satisfies the MVT}

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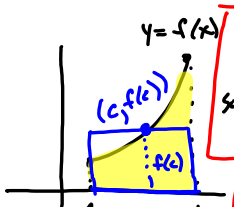
Mean Value Theorem for Definite Integrals p 288



If $f(x)$ is continuous on $[a,b]$ then there exists c in $[a,b]$ so that: $\int_a^b f(x)\,dx = (b-a)\,f(c)$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

mean value of f(x) (average)

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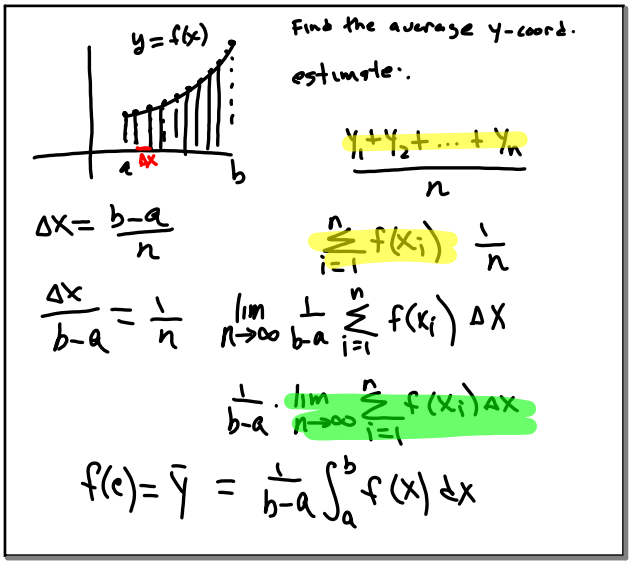
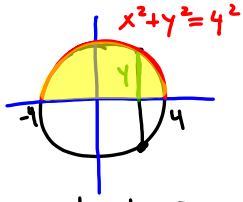


Nov 6-10:45 AM

How long is the average chord of a circle of radius 4? Find the value that satisfies the Mean Value Theorem for Definite Integrals. Find c



$y = \sqrt{16-x^2}$

$\bar{y} = \frac{1}{4-(-4)} \int_{-4}^{4} \sqrt{16-x^2}\,dx$

ave chord = 2π

$\bar{y} = \frac{1}{8} \cdot \frac{1}{2}\pi \cdot 4^2$

$\bar{y} = \pi$

To find c:

$\pi = \sqrt{16-x^2}$

$\pi^2 = 16-x^2$

$x^2 = 16-\pi^2$

$c = x = \pm\sqrt{16-\pi^2}$

Nov 13-5:31 PM